MOMENTUM

Definition: $\boxed{\vec{p} = m\vec{v}}$ → units are $kg \cdot m/s$

(no shortcut)

it's a vector!!

Momentum is important for a couple reasons: it's conserved and the more general form of Newton's Second Law

1) N2L: $\Sigma\vec{F} = m\vec{a}$ is actually a special case. Should be written as

$$\Sigma\vec{F} = \frac{d\vec{p}}{dt}$$

← really only need if doing rockets or special relativity

So $\Sigma F = \frac{d(m\vec{v})}{dt}$ → if m is constant

then $\Sigma\vec{F} = m\frac{d\vec{v}}{dt}$

$\Sigma\vec{F} = m\vec{a}$

2) Impulse: Let's just call "F" the net force for a moment:

$\therefore \quad \vec{F} = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = \vec{F}\,dt$

$\int d\vec{p} = \int \vec{F}\,dt$

$$\Delta\vec{p} = \int \vec{F}\,dt$$

→ 

Area = J

so Area = Δp

The integral is called IMPULSE and the impulse on an object is equal to the change in momentum of the object

Hey! kind of like $\int F \cdot dx = \Delta K$

momentum of the object.

$$\boxed{\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}}$$

units of Impulse are N·s or kg·m/s

The simple algebra version of this is

$$\boxed{J = \bar{F}t}$$

where $\bar{F}$ is the average force over the time interval t

3) N3L:

$$F_{AB} = -F_{BA}$$

2 objects interact

So $\quad F_{AB}\,dt = -F_{BA}\,dt$

So $\quad \int F_{AB}\,dt = -\int F_{BA}\,dt$

the impulses are equal and opposite

so $\quad \boxed{\Delta\vec{p}_B = -\Delta\vec{p}_A}$

Means momentum is conserved! If B gains momentum, then A loses the same amount and vice versa!

Let's rearrange that statement to make it easier to do problems:

$$\Delta \vec{p}_B = -\Delta \vec{p}_A$$

① $$\Delta \vec{p}_A + \Delta p_B = 0$$

i.e. the total change in momentum from the two object pushing each other is 0.

or

② $$\vec{p}_{B_f} - \vec{p}_{B_i} = -\left(\vec{p}_{A_f} - \vec{p}_{A_i}\right)$$

$$\vec{p}_{B_f} + \vec{p}_{A_f} = \vec{p}_{A_i} + \vec{p}_{B_i}$$

$$\sum \vec{p}_i = \sum \vec{p}_f$$

i.e. the total initial momentum is the same as the total final momentum.

This is the way I usually approach conservation of momentum.